Block (2D Semi-Discrete GLS3) Derivation:

II. Tidbits

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1. Change of Type

Numerical discretization errors often lead to change of type in a Maxwell-B fluid, leading to the appearance of non-physical characteristics [1, 2]. The necessary condition to prevent this change of type, is that the two additional stress tensors $T_A$ and $T_B$ remain positive-definite in the entire flow domain:

\[
T_A = T + \frac{\mu_1}{\lambda} I, \\
T_B = T_A - \rho v v,
\]

where $T$ is the viscoelastic stress tensor (also referred to as $T_1$ in Oldroyd-B fluid), $\mu_1$ is the VE viscosity, $\lambda$ is the relaxation time, and $vv$ is the rank-1 velocity tensor. For the creeping flow case normally considered, the rank-1 term is omitted, and $T_B = T_A$. In 2D, the eigenvalues of $T_A$ can be computed as follows:

\[
\det(T_A - sI) = \begin{vmatrix} T_{xx} + \frac{\mu_1}{\lambda} - s & T_{xy} \\ T_{xy} & T_{yy} + \frac{\mu_1}{\lambda} - s \end{vmatrix} = \left(T_{xx} + \frac{\mu_1}{\lambda} - s\right)\left(T_{yy} + \frac{\mu_1}{\lambda} - s\right) - T_{xy}^2
\]

\[
= s^2 - \left(T_{xx} + \frac{\mu_1}{\lambda} + T_{yy} + \frac{\mu_1}{\lambda}\right)s + \left(T_{xx} + \frac{\mu_1}{\lambda}\right)\left(T_{yy} + \frac{\mu_1}{\lambda}\right) - T_{xy}^2.
\]

The eigenvalues are obtained by setting the above determinant to 0, and solving the quadratic equation:

\[
\Delta = \left(T_{xx} + \frac{\mu_1}{\lambda}\right)^2 + 2\left(T_{xx} + \frac{\mu_1}{\lambda}\right)\left(T_{yy} + \frac{\mu_1}{\lambda}\right) + \left(T_{yy} + \frac{\mu_1}{\lambda}\right)^2
\]

\[-4\left(T_{xx} + \frac{\mu_1}{\lambda}\right)\left(T_{yy} + \frac{\mu_1}{\lambda}\right) + 4T_{xy}^2
\]

\[
= (T_{xx} - T_{yy})^2 + 4T_{xy}^2.
\]

\[
s_{1,2} = \frac{(T_{xx} + \frac{\mu_1}{\lambda}) + (T_{yy} + \frac{\mu_1}{\lambda}) \pm \sqrt{\Delta}}{2}.
\]

Both the eigenvalues $s_{1,2}$ must be positive.
2. Boundary Conditions

The constitutive equation can be written in using the index notation:

\[
\frac{1}{2\mu_1} T_{ij} + \frac{\lambda}{2\mu_1} u_k \partial_k T_{ij} - \frac{\lambda}{2\mu_1} (\partial_k u_i T_{kj} + T_{ik} \partial_k u_j) - \varepsilon_{ij} = 0. \tag{6}
\]

The fully-developed flow in the $x$-direction, defined as a parabolic distribution $u(y)$, will have the following extra-stress components:

\[
\begin{align*}
T_{xx} &= \tilde{T}_1 = 2\lambda \mu_1 \left( \frac{\partial u}{\partial y} \right)^2, \\
T_{xy} &= \tilde{T}_2 = \mu_1 \frac{\partial u}{\partial y}, \\
T_{yy} &= \tilde{T}_3 = 0, \\
\frac{\partial p}{\partial x} &= \frac{\partial T_{xy}}{\partial y}.
\end{align*}
\tag{7-10}
\]

History

November 21, 2001  Split from part I.

References
